A_1, A_3, and Y_1 necessary to obtain tolerable oxygen steady-state points. To deal with oxygen emergencies, we must add a safety margin to the amount of oxygen in storage.

To summarize, in order to have tolerable steady-state points, the lithium carbonate system being considered must be designed in such a fashion that the control variables B, Y_1, A_1, A_2, A_3 satisfy

$$A_{1\text{av}} \geq A_1{}^m \geq 6.05 \tag{18a}$$

$$\begin{aligned} &[1 - (b/10.9)]B_{\text{av}} + A_{1\text{av}}A_{2\text{av}} \leq 0.908 \\ &0.555 \leq [1 - (b/16.4)]B_{\text{av}} + A_{1\text{av}}A_{2\text{av}} \end{aligned} \tag{18b}$$

$$60.58 \leq (4.92)A_3A_{1\text{av}} + Y_{1\text{av}} \leq 62.31 \tag{18c}$$

Here the unit of time is taken to be one day, and we have simply rewritten Eqs. (14) and (17). The problem of weight considerations now becomes that of minimizing a weight function of the control variables under the restriction of (18). Equation (18) may be satisfied by eliminating the addended water separator unit (i.e., take $B = b = 0$) and choosing suitable values for A_2. In addition to (18), there are requirements on the system's ability to restore an intolerable atmospheric composition to a tolerable one. In a fashion indicated earlier, these are realized in the form of certain specified values for $A_1{}^M$, B^M, $Y_1{}^M$, etc. As a first approximation, we may write the weight of the lithium carbonate system in the form

$$\begin{aligned} W = C_1A_{1\text{av}} + C_2A_3A_{1\text{av}} + C_3A_1{}^M + \\ C_4B_{\text{av}} + C_5B^M + C_6Y_{1\text{av}}T_0 + \\ C_7T_0(Y_1{}^M - Y_{1\text{av}}) + C_8Y_{4\text{av}}T_0 + C_9T_0(Y_4{}^n - Y_{4\text{av}}) \end{aligned} \tag{19}$$

where T_0 is the total mission time, and the C_i's are constants that can be calculated from engineering data. The physical interpretation of the terms in (19) is given in Table 1. For a given mission time T_0, the optimum values for the control variables, from the weight point of view, in the lithium carbonate system may be found by minimizing W in (19) under the restriction in (18).

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Analysis of Stored Gas Pressurization Systems for Propellant Transfer

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The choice of a rocket propellant feed system is, by necessity, based on weight and reliability. Among the methods available for propellant transfer during a space mission with many vehicle maneuvers, the most attractive, primarily due to reliability and simple restart procedure, is expulsion by pressurization with stored, low molecular weight, inert gas. In this paper the performance analysis method for such a propellant transfer-system design is presented. Theoretical equations are developed from fundamental thermodynamic and heat-transfer concepts.

Nomenclature

A_E = heat-transfer area, ft^2
c_P = heat capacity at constant pressure, Btu/lbm-°R
c_V = heat capacity at constant volume, Btu/lbm-°R
d = differential operator
D = diameter, ft
g = acceleration, ft/sec^2
h = specific enthalpy, Btu/lbm
h_{fg} = latent heat of vaporization of the propellant, Btu/lbm
h_c = heat-transfer film coefficient, Btu/sec-ft^2-°R
J = mechanical equivalent of heat, 778.2 ft-lb/Btu
k = thermal conductivity, Btu-ft/sec-ft^2-°R
m = mass, lbm
Q = heat, Btu
P = pressure, lb/ft^2
R = specific gas constant, ft-lb/lbm-°R
T = temperature, °R
ΔT = difference between pressurant and bottle wall temperatures, °R
U = internal energy, Btu
U_e = overall heat-transfer coefficient, Btu/sec-ft^2-°R
V = volume, ft^3
$\bar{V}$ = mean velocity, fps
Z = compressibility factor
β = coefficient of thermal expansion, 1/°R
π = partial pressure, lb/ft^2
ρ = density, lbm/ft^3
μ = viscosity, lbm/ft-sec
θ = time, sec

Subscripts

B = storage bottle
C = coil
E = heat exchanger
G = pressurant
I = inside

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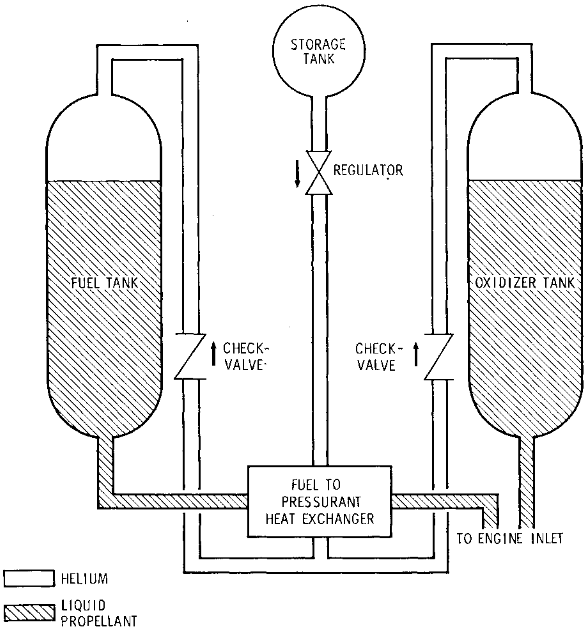

Fig. 1 Propellant system.

M = weighted term as defined in text
O = outside
P = propellant
PG = propellant gas vapor
PL = propellant liquid
U = propellant tank ullage

Introduction

IN a typical space vehicle the propellant is expelled in numerous engine firing periods interspersed between coast phases throughout the mission. In several current applications, minimum vehicle weight is achieved utilizing an engine with a chamber pressure of approximately 100 psia. To achieve maximum reliability, propellants are fed to the engine from pressurized storage tanks. For man-rated vehicles, mission success or reliability becomes the overriding selection criterion. From this perspective, storable propellants are relatively insensitive to space environment, orientation with respect to the sun, and amenable to simple design due to high density and low temperature control requirements. The storables compare favorably with higher performance but bulkier cryogenics on vehicle payload capability. This becomes more apparent after considering the requirements for a man-rated vehicle for structural rigidity and meteorite protection, along with the problems associated with solubility of pressurant in propellants, propellant loss due to vaporization, and zero gravity venting.

During a typical interplanetary mission, the spacecraft propulsion system is employed in various space maneuvers. When the engine is firing, the inert pressurant gas that is stored initially at high pressure (2000–5000 psia) and ambient temperature, is allowed to expand to a regulated pressure. The gas ultimately flows into the propellant tank ullage and drives the propellant to the rocket engine. As pressurant flows out of the high pressure bottle, the temperature and pressure in the bottle follow a polytropic expansion path that can be analytically predicted. The gas entering the ullage does not attain propellant-vapor equilibrium during this period. The degree to which propellant vaporization occurs depends upon the gas mixing in the ullage, the proximity of liquid surface to the entering gas, and the magnitude of liquid-gas vapor pressure and temperature difference as the driving force in mass and heat transfer.

During the comparatively long periods when the engine is not firing, i.e., the coast periods, the tank ullage reaches thermal and vapor pressure equilibrium. In these periods, the ullage pressure rises due to heat and mass transfer from the liquid propellant. This overpressure results in an increase in the tank design pressure and, consequently, an increase in tank weight. Also, operation of the engine at higher than normal propellant supply pressures may be objectionable.

The rise in ullage pressure can be minimized by the addition of heat to the pressurant. A propellant to pressurant heat exchanger of reasonable size in the propellant feed lines will warm the pressurant to a temperature within a few degrees of the propellant temperature, and thus minimize the rise in ullage pressure.

The method of analysis for such a propellant system must provide a complete history of system operation during the entire space mission. The development of the analytic method is presented below. The operating ranges over which the method is applicable are:

Initial pressurant pressure, lb/in^2	2000–5000
Ullage pressure, lb/in^2	20–300
Chamber pressure, lb/in^2	10–200
Ullage temperature, °R	400–600

These ranges do not necessarily limit the usefulness of the analytical method, but some modification would be needed with respect to ullage heat transfer for cryogenics.

Analysis

A simplified schematic of the pressure-fed propellant system is presented in Fig. 1. In order to facilitate the analysis, the following systems of the pressurization system are considered: 1) pressure storage bottle, 2) heat exchanger, and 3) propellant tank ullage. In the analysis of each subsystem, it is assumed that the gases are ideal except for the gas in the pressurant storage bottle. For the stored pressurant, the experimental compressibility factor is included in the equation of state to correct for gas nonideality.[1]

Pressurant Storage Bottle Analysis

Figure 2 illustrates the system analyzed. The thermodynamic system, as indicated by the boundaries of the envelope, is the volume occupied by the pressurant gas within the storage bottle. During the period of time $d\theta$ the following thermodynamic process occurs: 1) the gas within the thermodynamic system experiences a change in internal energy dU; 2) heat is transferred from the storage bottle wall into the system dQ; and 3) enthalpy is lost from the system $(c_P)_G T_G dm_G$. The energy balance expressed in differential form is

$$dU = dQ - (c_P)_G T_G dm_G \tag{1}$$

where

$$dU = d[(c_V)_G P_G V_B / R_G Z_G] \tag{2}$$

The total derivative, with $(c_V)_G/R_G$ assumed to be constant

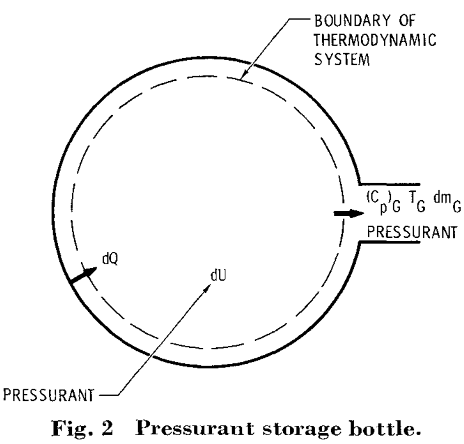

Fig. 2 Pressurant storage bottle.

and with volume V_B constant, is equal to the change in internal energy:

$$dU = -\frac{(c_V)_G P_G V_B}{R_G Z_G{}^2}\, dZ_G + \frac{(c_V)_G V_B}{R_G Z_G}\, dP_G \qquad (3)$$

The heat transfer from the gas storage bottle wall was found experimentally to agree closely with the heat transfer from vertical tubes and plates by natural convection. According to Stoever,[2] the heat-transfer coefficient is determined by the equation

$$(h_c)_B = 0.128 \left(\frac{k^2\rho^2 c_P \beta g \Delta T}{\mu}\right)_G^{1/3} \qquad (4)$$

The total heat transfer during $d\theta$ is

$$dQ = (h_c)_B A_B \Delta T d\theta \qquad (5)$$

Simultaneous solution of Eqs. (1) and (3) for dP_G and simplification by substituting the equivalent terms as defined by the modified gas equation give the following differential equation:

$$dP_G = \frac{P_G}{(c_V)_G m_G T_G}\, dQ + \frac{P_G}{Z_G}\, dZ_G - \frac{(c_P)_G P_G}{(c_V)_G m_G}\, dm_G \qquad (6)$$

Equation (6) can be interpreted as the sum total of the change in pressure due to the heat transfer into the gas, the change in the compressibility factor with temperature and pressure, and the flow of mass out of the system, respectively. Because of the heat-transfer term, a trial and error solution of this equation is required which is based on the quantity of gas that is removed. This is necessitated by the results of the analysis of the heat exchanger and the propellant tank ullage.

Heat Exchanger

Only a brief treatment of the heat exchanger analysis is given here. For an example, a low-pressure coil type of heat exchanger, in which the liquid propellant flow is normal to the axis of the coils, is analyzed.

The rate of sensible heat rise of the gas is equated to the heat-transfer rate from liquid to gas. The temperature difference of the liquid at the inlet and outlet is insignificant. The analysis gives the following equation, which is based upon the over-all heat-transfer coefficient and the average heat exchanger area:

$$T_E = T_{PL} - \frac{T_{PL} - T_G}{\exp\{U_e A_E / [(c_P)_G (dm_G/d\theta)]\}} \qquad (7)$$

The wall to gas heat-transfer coefficient as given by Stoever[3] is

$$(h_c)_G = \frac{k_G}{D_I}\, 0.0225 \left(\frac{D_I \bar{V} \rho}{\mu}\right)_G^{0.8} \left(\frac{c_P \mu}{k}\right)_G^{0.4} \left(1 + 3.5\, \frac{D_I}{D_c}\right) \qquad (8)$$

when the coil correction is included.[4] The liquid to wall heat-transfer coefficient, as given by Obert,[5] for a bank of tubes greater than ten rows is

$$(h_c)_{PL} = \frac{k_{PL}}{D_O}\, 0.264 \left(\frac{D_O \bar{V} \rho}{\mu}\right)_{PL}^{0.6} \left(\frac{c_P \mu}{k}\right)_{PL}^{0.333} \qquad (9)$$

This heat-transfer coefficient was increased by 20% due to the increased turbulence caused by the coils. This correction is comparable to the adjustment made for heating or cooling of liquids inside coils. From these coefficients and the thickness and conductivity of the wall, the over-all heat-transfer coefficient U_e is calculated.

Propellant Tank Ullage

The ullage thermodynamic system during acceleration is defined as the ullage of each of the propellant tanks. This system is illustrated in Fig. 3. Each propellant tank contains a mixture of inert gas pressurant and vaporized propellant. The equations are developed for general case of either fuel or oxidizer. For the over-all analysis, specific propellant properties are used in each propellant tank.

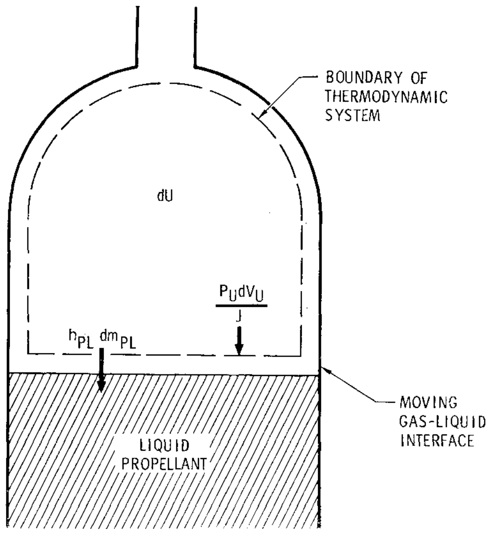

Fig. 3 Propellant tank ullage (transient pressure period).

During the coast period, thermal and vapor pressure equilibrium is attained between propellant and pressurant. The ullage pressure for the subsequent firing initially will be above the regulator output pressure, due to heat and mass transfer into the gas. The propellant will therefore, be expelled with no pressurant entering the propellant tank. During this pressure decay period, the ullage is expanding with over-saturation assumed to be relieved by vapor condensation along the equilibrium line. The heat of condensation is "taken up" by the ullage system, the condensate is considered to leave the thermodynamic system, and the heat transfer from tank walls and liquid surface is negligible during short firing durations with small temperature differences between the liquid and the gas.

The following thermodynamic process occurs during over-pressure relief period: 1) the gas in the ullage system experiences a change in internal energy dU; 2) work is done by the system in expelling the propellant $(P_U/J)dV_U$; 3) enthalpy is lost from the system $h_{PL}dm_{PL}$. For this open system, the energy balance by the first law of thermodynamics is expressed by the following differental equation:

$$dU = -(P_U/J)dV_U - h_{PL}dm_{PL} \qquad (10)$$

The change in the system internal energy dU can be related to the change in the system enthalpy by the following equation:

$$dU = dH - (P_U/J)dV_U - (V_U/J)dP_U \qquad (11)$$

By considering ideal mixing,

$$dH = (c_P m)_M dT + h_{PG} dm_{PG} \qquad (12)$$

where $(c_P m)_M = (c_P)_G m_G + (c_P)_{PG} m_{PG}$. The terms dU and dH are removed by simultaneous solution of Eqs. (10–12):

$$(c_P m)_M dT_U + h_{PG} dm_{PG} + h_{PL} dm_{PL} = (V_U/J) dP_U \qquad (13)$$

Since

$$dm_{PG} = -dm_{PL} \qquad (14)$$

and

$$h_{fg} = h_{PG} - h_{PL} \text{ (heat of vaporization, Btu/lbm)} \qquad (15)$$

the following differential equation is derived in which dm_{PG}, dT_U, and dP_U are considered unknowns:

$$h_{fg} dm_{PG} + (c_P m)_M dT_U = (V_U/J) dP_U \qquad (16)$$

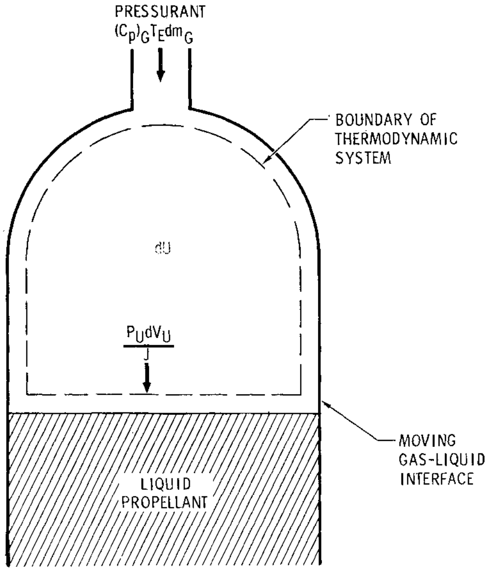

Fig. 4 Propellant tank ullage (normal firing period).

A second differential equation is developed by taking the total derivative of the ideal gas relation, $m_{PG} = \pi_{PG}V_U/R_PT_U$. The ideal gas equation is considered applicable because of the low operating pressures in the tank ullage (20–300 psia). Since the vapor pressure can be expressed as a function of the temperature, $d\pi_{PG} = f'(T_U)dT_U$, one obtains by substitution

$$dm_{PG} + \left[\frac{\pi_{PG}V_U}{R_PT_U{}^2} - \frac{V_Uf'(T_U)}{R_PT_U}\right]dT_U = \frac{\pi_{PG}}{R_PT_U}dV_U \quad (17)$$

A third differential equation is found by obtaining the total derivative dP_U, assuming that the gas mixture is ideal:

$$dP_U = \frac{T_U}{V_U}d(Rm)_M + \frac{(Rm)_M}{V_U}dT_U - \frac{(Rm)_MT_U}{V_U{}^2}dV_U \quad (18)$$

where $(Rm)_M = R_{PG}m_{PG} + R_Gm_G$ and $d(Rm)_M = R_Pdm_{PG}$.

Equations (16–18) are solved simultaneously for the unknown differentials dP_U and dT_U by using determinants. This operation gives

$$dP_U = \frac{\left[\frac{(c_Pm)_M}{m_{PG}}(P_U - \pi_{PG}) + \frac{P_Uf'(T_U)h_{fg}}{\pi_{PG}}\right]\frac{dV_U}{V_U}}{\left[\frac{1}{T_U} - \frac{f'(T_U)}{\pi_{PG}}\right]\left[h_{fg} - \frac{R_PT_U}{J}\right] + \frac{1}{m_{PG}}\left[\frac{(Rm)_M}{J} - (c_Pm)_M\right]} \quad (19)$$

$$dT_U = \frac{\left\{h_{fg} - \frac{T_U}{J}\left[R_P - \frac{(Rm)_M}{m_{PG}}\right]\right\}\frac{dV_U}{V_U}}{\left[\frac{1}{T_U} - \frac{f'(T_U)}{\pi_{PG}}\right]\left[h_{fg} - \frac{R_PT_U}{J}\right] + \frac{1}{m_{PG}}\left[\frac{(Rm)_M}{J} - (c_Pm)_M\right]} \quad (20)$$

From the changes in pressure and temperature, as determined by Eqs. (19) and (20), the final conditions following an incremental change in volume are calculated by using the equation of state.

During the normal firing period, i.e., when the ullage pressure is equal to the regulator output pressure, helium gas flows into the propellant tanks to displace the liquid propellant. For this period, it is assumed that heat transfer from the tank wall and heat and mass transfer from the liquid are negligible. The thermodynamic system is illustrated in Fig. 4. The following thermodynamic process occurs: 1) the gas experiences a change in internal energy dU; 2) enthalpy enters the system in the form of gas at the heat exchanger exit temperature $(c_P)_GT_Edm_G$; and 3) work is done by the system in expelling the propellant $(P_U/J)dV_U$.

The energy balance of the system is

$$dU = -(P_U/J)dV_U + (c_P)_GT_Edm_G \quad (21)$$

Since the pressure remains constant, the change in internal energy is given by

$$dU = P_UV_Ud(c_V/R)_M + (c_V/R)_MP_UdV_U \quad (22)$$

Determination of $d(c_V/R)_M$ is made by differentiating the ratio of the weighted average c_V and R, with the change in mass of the vaporized propellant assumed to be zero, i.e.,

$$(c_V/R)_M = \frac{(c_V)_{PG}m_{PG} + (c_V)_Gm_G}{R_Pm_{PG} + R_Gm_G} \quad (23)$$

$$d(c_V/R)_M = \frac{[(c_V)_G - (c_V/R)_MR_G]}{R_Pm_{PG} + R_Gm_G}dm_G \quad (24)$$

Equations (21, 22, and 23) are solved simultaneously for the incremental amount of gas that flows into the tank to support a corresponding change in ullage volume during an increment of time:

$$dm_G = \frac{[(c_V/R)_M + 1/J]P_UdV_U}{(c_P)_GT_E - [(c_V)_G - (c_V/R)_MR_G]T_U} \quad (25)$$

Over-All Analysis

The over-all analysis of the pressurization system is made by employing the equations just developed, along with the equations-of-state of the compounds in the system, and solving the set by numerical methods on a digital computer.

A program was developed for a system that utilizes helium gas for the pressurant, nitrogen tetroxide (N_2O_4) for the oxidizer, and 50% unsymmetrical di-methyl hydrazine and 50% hydrazine (Aerozine-50) for the fuel. The propellant system is insulated such that there is no heat transfer from an external source, but there is radiant heat transfer from the propellant tank to the helium storage bottle. Since the engine burning period is small compared to the coasting period, this radiant flux is neglected during the burning period. At the beginning of the first firing period, the ullage gas is assumed to be in equilibrium at the regulator output pressure (there is no transient pressure period). For the succeeding firing periods, the coast period is sufficiently long to allow total saturation of the ullage at the liquid temperature and, therefore, causes an overpressure, depending upon the conditions at the end of the preceding firing period.

Data from a computer run are presented to illustrate schematically the variation in the thermodynamic properties of

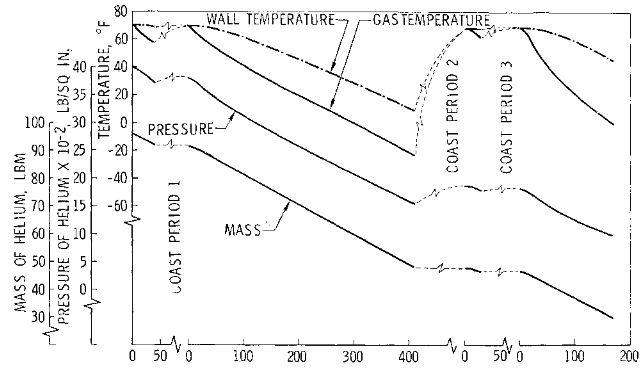

Fig. 5 Pressurant storage bottle—variation of temperature, pressure, and mass.

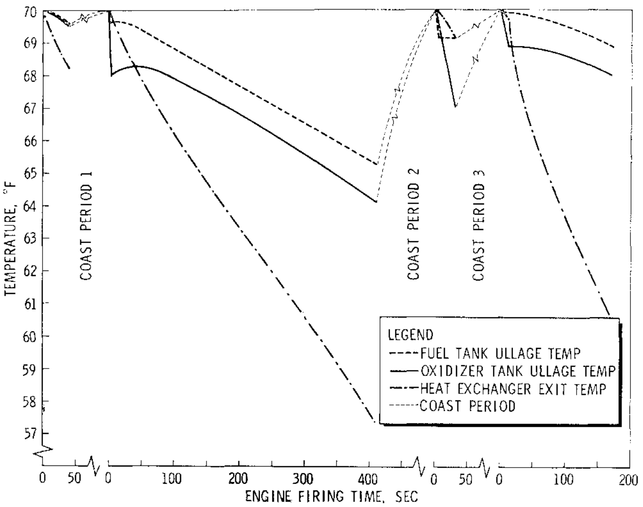

Fig. 6 Propellant tank ullage—variation of temperature.

the propellant transfer system (Figs. 5, 6, and 7). The thermodynamic properties of the gases in the storage bottle and propellant tank ullage are given for an entire flight, consisting of engine operation at ullage overpressure conditions and during operation with regulated ullage pressures. In order to obtain an illustrative example, an engine firing period that is not sufficiently long to allow the ullage pressure to drop to the regulated steady-state tank ullage pressure is included.

The variation of the temperature, pressure, and mass of the pressurant in the storage bottle, together with the variation of the storage bottle wall temperature, is shown in Fig. 5. The gas is initially at 530°R and 400 psia in two spheres of about 20 ft^3 each. The broken lines indicate temperature and pressure rise during the coast periods. The corresponding variation in the propellant tank ullage temperature, along with the heat exchanger exit temperature, are presented in Fig. 6. This curve indicates the rapid drop in temperature that occurs during the period in which the tank ullage pressures are transient. During this period, the temperature is dictated by the expansion process with no helium entering the propellant tank. Following the transient pressure period, the temperature drop is moderate as required by the heat exchanger exit temperature. The over-all drop, however, is comparatively small, on a percentage basis, and has, therefore, little influence on the ullage pressure rise during the coast period.

The magnitude of the pressure rise during the coast period and the variation of the tank ullage pressure and temperature with engine firing time is illustrated in Fig. 7. The curves represent only the initial phase of the three engine firing periods. The rise in pressure is due primarily to propellant vaporization during the coast period. Since the vapor pressures of the propellants differ, the tank ullage pressures of the fuel and oxidizer are markedly different. The curves also illustrate the decrease in the rate of pressure drop in each succeeding engine firing period. The rate is a function of the ullage volume. In addition, during a short engine firing period, the ullage pressure may not drop to the regulator output pressure. Therefore, there will be comparatively small rise in the tank ullage pressure during the succeeding firing period.

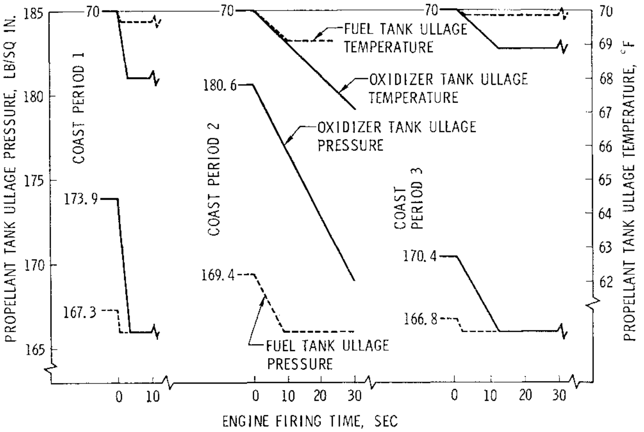

Fig. 7 Propellant tank ullage—variation of temperature and pressure.

Conclusions

The thermodynamic analysis of a gas pressurization system for propellant transfer to the rocket engine can be performed with some degree of confidence. The analytic derivations can be best solved with a high-speed computer. The equations that are derived can be used for different pressurant/propellants combinations and comparable gas pressurization systems. However, the difference in each system must be recognized and included in the analysis.

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